ISSN 1870-4069

Interference Phenomena and Photon Statistics in a Cross Cavity

Ana Cristina Uribe-Chairez[1], José Manuel González-Vargas[1], Lizbeth Romero-Lira[1], José Javier Sánchez-Mondragón[2], Néstor Lozano-Crisóstomo[1], Julio César García-Melgarejo[1]

[1] Universidad Autónoma de Coahuila, Facultad de Ingeniería Mecánica y Eléctrica, Mexico

[2] Instituto Nacional de Astrofísica, Óptica y Electrónica, Departamento de Óptica, Mexico

julio.melgarejo@uadec.edu.mx

Abstract. We discuss the Jaynes-Cummings interactions of a two-level atom in a two-dimensional perfect cavity. Quantum superpositions in our cross-cavity configuration produce interference phenomena, which manifest as collapses and revivals in unforeseen cases by the Jaynes-Cummings Model. We also compute numerically the Mandel parameter and the second-order two-time correlation function and verify the existence of interference-improved sub-Poissonian light and photon antibunching.

Keywords: Jaynes and cummings model, novel cavity, quantum interference, antibunching.

1 Introduction

The Jaynes-Cummings Model (JCM) is one of the most important pillars of modern quantum optics, especially of the cavity quantum electrodynamics. The JCM describes the interactions between an atom and the quantized field of a perfect one-dimensional cavity [6, 2]. The study of the JCM interactions allowed the theoretical prediction of the so-called collapses and revivals that together with its experimental corroboration were a proof of the quantum nature of the light [4, 8]. Different aspects of the collapses and revivals have been discussed over the past decades; however, they are still interpreted as the result of interference between the interaction of the atom and each one of the occupation states of the cavity field.

2 Mathematical Model

Our model describes the interaction of a two-level atom with the cavity field of two perfect cavities oriented along x and y. The Hamiltonian of our system, under dipole and rotating wave approximation, is $H = H_0 + H_{\text{int}}$.

The free and the interaction components are given, respectively, by:

$$H_0 = \frac{1}{2}\hbar\omega_0\sigma_z + \hbar\omega\left(a^\dagger a + b^\dagger b\right), \tag{1}$$

$$H_{\text{int}} = \hbar\lambda_x\left(a\sigma_+ + a^\dagger\sigma_-\right) + \hbar\lambda_y\left(b\sigma_+ + b^\dagger\sigma_-\right). \tag{2}$$

In Equations (1)-(2), we denote the atomic transition frequency by ω_0 and assume that both cavities have the same frequency ω; in addition, λ_x and λ_y are the coupling constants along x and y that we will take as real quantities. The field operators a and b describe the field in our configuration, they satisfy the standard commutation rules of two independent quantum oscillators. On the other hand, the lowering $\sigma_- = |g\rangle\langle e|$ and raising $\sigma_+ = |e\rangle\langle g|$ operators govern the transitions between the excited $|e\rangle$ and the ground $|g\rangle$ states, and they satisfy the commutation rule $\sigma_z = [\sigma_+, \sigma_-]$.

The cross-cavity configuration provides the possibility of adjusting its coupling constants by changing geometrical parameters such as the geometrical orientation of the atomic dipole or the volume of each arm.

In the light of that, we define an effective coupling constant $\lambda_{\text{eff}} = \sqrt{\lambda_x^2 + \lambda_y^2}$ and the coupling parameter by $\theta = 2\arctan\left(\lambda_y/\lambda_x\right)$. The coupling constants can be conveniently written as $\lambda_x = \lambda_{\text{eff}}\cos(\theta/2)$ and $\lambda_y = \lambda_{\text{eff}}\sin(\theta/2)$. The coupling parameter is not only an important parameter that dictates how strongly the atomic dipole couples to each arm of the cross cavity, but also it allows defining the new field operators through $A = \cos(\theta/2)a + \sin(\theta/2)b$ and $B = -\sin(\theta/2)a + \cos(\theta/2)b$. The new field operators satisfy the algebra of two independent harmonic oscillators and the total number operator $a^\dagger a + b^\dagger b = A^\dagger A + B^\dagger B$ is preserved.

3 Quantum Superpositions in the Cross-Cavity

By rewriting the Hamiltonian in terms of A and B emerges a mathematical structure similar to the Jaynes-Cummings Hamiltonian, which allows us to describe the dynamics of the cross-cavity through the state:

$$|\psi(t)\rangle\rangle = \sum_{MN}\left[\phi_{MN,e}(t)|e\rangle|M,N\rangle\rangle + \phi_{MN,g}(t)|g\rangle|M+1,N\rangle\rangle\right]. \tag{3}$$

The analytical expression of the complex amplitudes $\phi_{MN,e}(t)$ and $\phi_{MN,g}(t)$ will be published elsewhere. A more interesting aspect of the state 3 is that it is given in terms of the two-quasi-mode Fock states (Denoted by double angle), they can be conveniently discussed in terms of the angular momentum formalism.

For that purpose, let us introduce the Schwinger rotation operator $S_y(\theta) = e^{-i\theta L_y}$ and the angular momentum operator $L_y = -i\left(a^\dagger b - b^\dagger a\right)/2$. By recognizing the equality $|0,0\rangle = |0,0\rangle\rangle$, the two-quasi-mode Fock states are obtained by a simple rotation $|M,N\rangle\rangle = S_y(\theta)|M,N\rangle$. Furthermore, the coefficients are obtained straightforwardly by:

$$|j,m\rangle\rangle = S_y(\theta)|j,m\rangle = \sum_{m'} d^j_{m',m}(\theta)|j,m'\rangle. \tag{4}$$

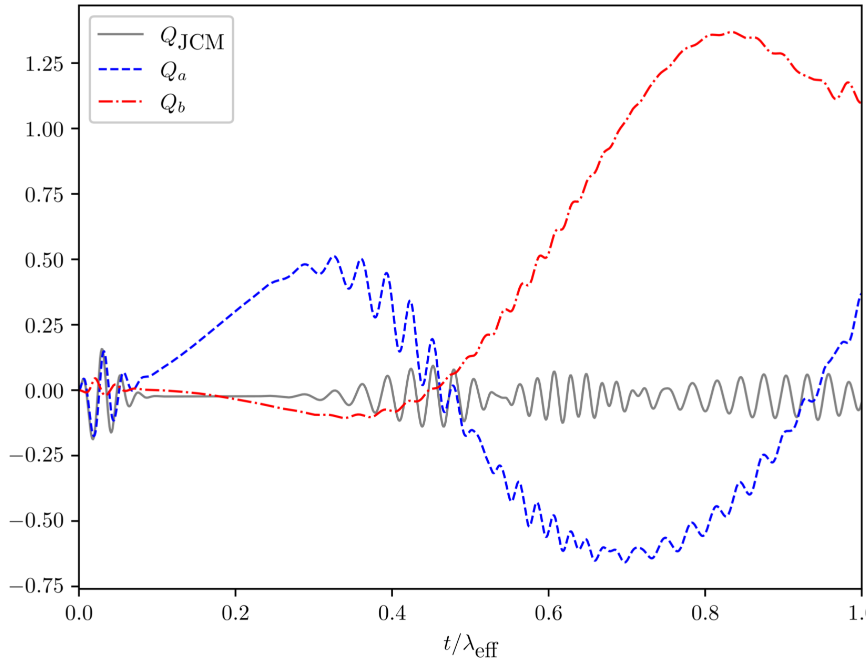

Fig. 1. Numerical computation of the Mandel parameter in the Jaynes-Cummings Model $Q_{\rm JCM}$ and in the horizontal Q_a and vertical Q_b arm of the cross-cavity. The initial state of the system is $|\alpha, 0, e\rangle$, with $|\alpha|^2 = 10$, and the coupling parameter is $\theta = \pi/4$. Notice that while the JCM predicts $Q_{\rm JCM} \sim -1/4|\alpha|^2$ in the regions between collapse and revival, the value of the Mandel parameter Q_a is considerably lower, i.e., it is considerably more sub-Poissonian.

In Eq. 4 we identify the angular momentum states $\left|j, m\right\rangle$, with quantum numbers $j = (M+N)/2$ and $m = (M-N)/2$. Moreover, the coefficients of the expansion are the elements of the well known Wigner d-matrix $d^{j}_{m',m}(\theta) = \left\langle j, m'\right| S_y(\theta) \left|j, m\right\rangle$ [9]. According to Eq. 4, the transformed angular momentum can be interpreted as the superposition of angular momenta over all possible values of m, and a constant value of j.

The transformation that we introduced to define new field operators in the cross-cavity also can be found in the quantum mechanical theory of a lossless Beam Splitter (BS). The field at the output ports of the BS are related to the field at the input ports through a Schwinger transformation, and the photon statistics for notable inputs is well-known [1]. From the wave function, some important quantities can be analytically evaluated such as the atomic inversion and the average number of photons. Our previous research shows that collapses and revivals may occur in unforeseen cases by the JCM [5].

4 Sub-Poissonian Light and Photon Antibunching

To discuss the photon statistics of the field in the cross cavity, we compute numerically the Mandel parameter and the second-order normalized correlation function. These quantities are given by:

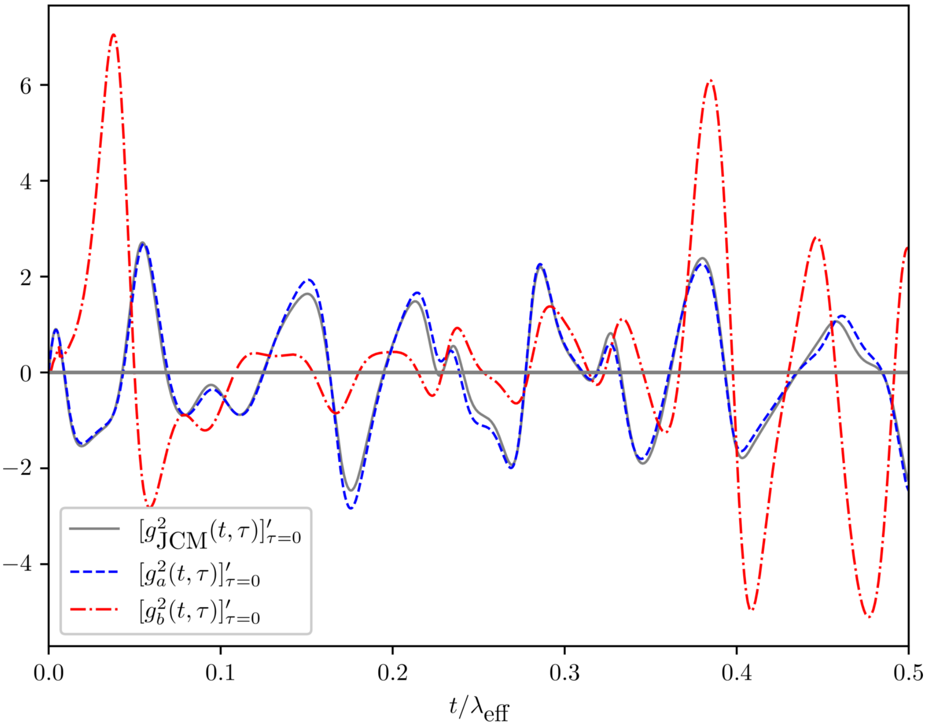

Fig. 2. Numerical computation of the derivative of the second-order correlation $g_O(t,\tau)$ as a function of τ in the Jaynes-Cummings Model and in the horizontal Q_a and vertical Q_b arm of the cross-cavity. The initial state of the system is $|\alpha,0,e\rangle$, with $|\alpha|^2 = 1$, and the coupling parameter is $\theta = \pi/8$. Notice that for this configuration, the antibunching properties of the JCM and the horizontal arm of the cross-cavity looks quite similar; on the other hand, there is a transference of photons towards the vertical arm with antibunching exhibiting an oscillatory behavior.

$$\begin{aligned} Q_O(t) &= \frac{\left\langle O^\dagger(t)\,O^\dagger(t)\,O(t)\,O(t)\right\rangle - \left\langle O^\dagger(t)\,O(t)\right\rangle^2}{\left\langle O^\dagger(t)\,O(t)\right\rangle}, \\ g_O^{(2)}(t,t+\tau) &= \frac{\left\langle O^\dagger(t)\,O^\dagger(t+\tau)\,O(t+\tau)\,O(t)\right\rangle}{\left\langle O^\dagger(t+\tau)\,O(t+\tau)\right\rangle\left\langle O^\dagger(t)\,O(t)\right\rangle}, \end{aligned} \tag{5}$$

where $O = a,\ b$. The numerical solutions were obtained withe the aid of the Python QuTiP Toolbox [7]. Let us recall that light whose photon number fluctuations are smaller than those of the Poisson distribution it is said sub-Poissonian and that a good measure of those fluctuations is the Q-parameter.

In figure 1, we show an striking numerical result of Mandel parameter in the cross cavity. Whereas the JCM predicts a value $Q_{\mathrm{JCM}} \sim -1/4\bar{n}$ between collapse and revival ($\bar{n}$ indicates the average of the photon distribution), the cross-cavity produces Mandel parameters along x and y considerably more negative; however, it seems that there is no a quasi-steady behavior in the short-term regime.

Another interesting statistical feature of the light is the degree of second-order coherence, which plays a central role in the definition of photon-antibunching. A widely accepted definition of antibunching occurs when the derivative of the normalized function $g_O^{(2)}(t, t+\tau)$ as a function of the delay time τ at $\tau = 0$ is positive [3]. Our numerical simulation focuses on weak field $|\alpha|^2 = 1$, and the results are presented in the figure 2.

5 Conclusions

The cross-cavity configuration that we presented is a tripartite system that allows inquiring on quantum superposition effects unforeseen in the bipartite JCM.

Acknowledgments. Supported by Universidad Autónoma de Coahuila and by CONACyT through the scholarships assigned to L. Romero-Lira (CVU:1104357) and J. M. González-Vargas (CVU:972195).

References

1. Campos, R. A., Saleh, B. E. A., Teich, M. C.: Quantum-mechanical lossless beam splitter: SU(2) symmetry and photon statistics. Physical Review A, vol. 40, pp. 1371–1384 (1989) doi: 10.1103/PhysRevA.40.1371
2. Cummings, F. W.: Stimulated emission of radiation in a single mode. Physical Review, vol. 140, no. 4A, pp. A1051–A1056 (1965) doi: 10.1103/PhysRev.140.A1051
3. Dung, H. T., Shumovsky, A. S., Bogolubov, N. N.: Antibunching and sub-poissonian photon statistics in the Jaynes-Cummings model. Optics Communications, vol. 90, no. 4-6, pp. 322–328 (1992) doi: 10.1016/0030-4018(92)90285-y
4. Eberly, J. H., Narozhny, N. B., Sanchez-Mondragon, J. J.: Periodic spontaneous collapse and revival in a simple quantum model. Physical Review Letters, vol. 44, no. 20, pp. 1323–1326 (1980) doi: 10.1103/PhysRevLett.44.1323
5. Garcia-Melgarejo, J. C., Lozano-Crisostomo, N., Rodriguez-Montero, P., Sanchez-Mondragon, J. J.: Dynamics of a two-level atom in a double perpendicular cavity. Photonics Letters of Poland, vol. 7, no. 2, pp. 35–37 (2015) doi: 10.4302/photon.lett.pl.v7i2.553
6. Jaynes, E., Cummings, F.: Comparison of quantum and semiclassical radiation theories with application to the beam maser. In: Proceedings of the IEEE, vol. 51, pp. 89–109 (1963) doi: 10.1109/proc.1963.1664
7. Johansson, J. R., Nation, P. D., Franco, N.: QuTiP 2: A Python framework for the dynamics of open quantum systems. Computer Physics Communications, vol. 184, no. 4, pp. 1234–1240 (2013) doi: 10.1016/j.cpc.2012.11.019
8. Rempe, G., Walther, H., Klein, N.: Observation of quantum collapse and revival in a one-atom maser. Physical Review Letters, vol. 58, no. 4, pp. 353–356 (1987) doi: 10.1103/physrevlett.58.353
9. Wigner, E.: Group theory: And its application to the quantum mechanics of atomic spectra. Elsevier (1952)